



Reconstruction of the 1801 Discovery Orbit of Ceres via Contemporary Angles-Only Algorithms

*A Presentation to the General Meeting of the
Denver Astronomical Society
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- **My goal with this presentation is to tell you about these three topics:**
 - The annual AMOS conference held in the fall each of year in Maui, Hawaii, and about what my wife, Karen, and I experienced there
 - The poster that I presented there
 - The technical content of the paper that I wrote to accompany my poster
- **Please note:**
 - I will talk about the poster and paper first, then about the AMOS conference itself
 - I will have a brief slide show at the end of my presentation
 - There should be time for questions/answers after the slide show
 - Please ask questions then



Presentation agenda follows poster panels in handout poster:

1. Path of Ceres during 1801 - 1802
2. Timeline 1801 - 1802
3. Timeline 1802 - Present
4. The Contemporary Algorithms
5. Flow of Solutions and Comparisons
6. Summary of Findings
7. The annual AMOS conference
8. Brief slide show
9. Question and answer session

Reconstruction of the 1801 Discovery Orbit of Ceres via Contemporary Angles-Only Algorithms

Path of Ceres from 1801 January 1 to 1806 May 23. This figure shows what the path of Ceres, as a dwarf planet in the asteroid belt, looked like from the date of Ceres's discovery by Piazzi through its observation by Olbers, Harding, and Bessel in 1805-1806.

Timeline 1801-1802

- Jan 1 through Feb 11 – Piazzi discovers/observes the first asteroid, Ceres.
- September – von Zach publishes Piazzi's observations in *Monatliche Correspondenz*. See ①
- October-November – von Zach publishes orbital analyses by Burckhardt, Olbers, and Piazzi.
- December – von Zach publishes orbital elements and search ephemeris of Gauss. Recovers Ceres on night of 1801 Dec 31 – 1802 Jan 1 using Gauss search ephemeris. See ②

Timeline 1802-Present

- 1840s – Star catalogs refined/standardized, making more accurate observations possible.
- 1895 – USNO's Simon Newcomb publishes highly accurate *Tables of the Sun*.
- 1950s – 1980s – Digital computers and atomic clocks developed and refined; J2000 reference frame adopted for orbital motion and FK5 star catalog reference frame.
- 1990s – Present – CCD imaging in telescopes; USNO star catalogs enable sub-arc-second angle measurements by telescopes using CCDs.

Flow of Solutions and Comparisons

The Contemporary Algorithms

Der IOD (Initial Orbit Determination)

- 3 angles-only (RA, DEC) observations
- Long arcs admissible, up to one orbital revolution
- Solves for range via 8th degree polynomial
- Epoch of elements is at middle observation time

HGM IOD – Heliocentric motion

- n angles-only (RA, DEC) observations
- n ≥ 3 but subject to short-arc limitation
- Iterates on range guesses for first and last observations
- Epoch of elements is at first ob time

ORBIT2 Numerical Integration

- Numerically integrates equations of motion of major Solar System planets and "Spare"
- "Spare" can be space probe, comet, asteroid, or dwarf planet
- Starts with DE405 initial state vectors of planets and J2000 initial state vector for "Spare"

Batch UPM DC – Heliocentric motion

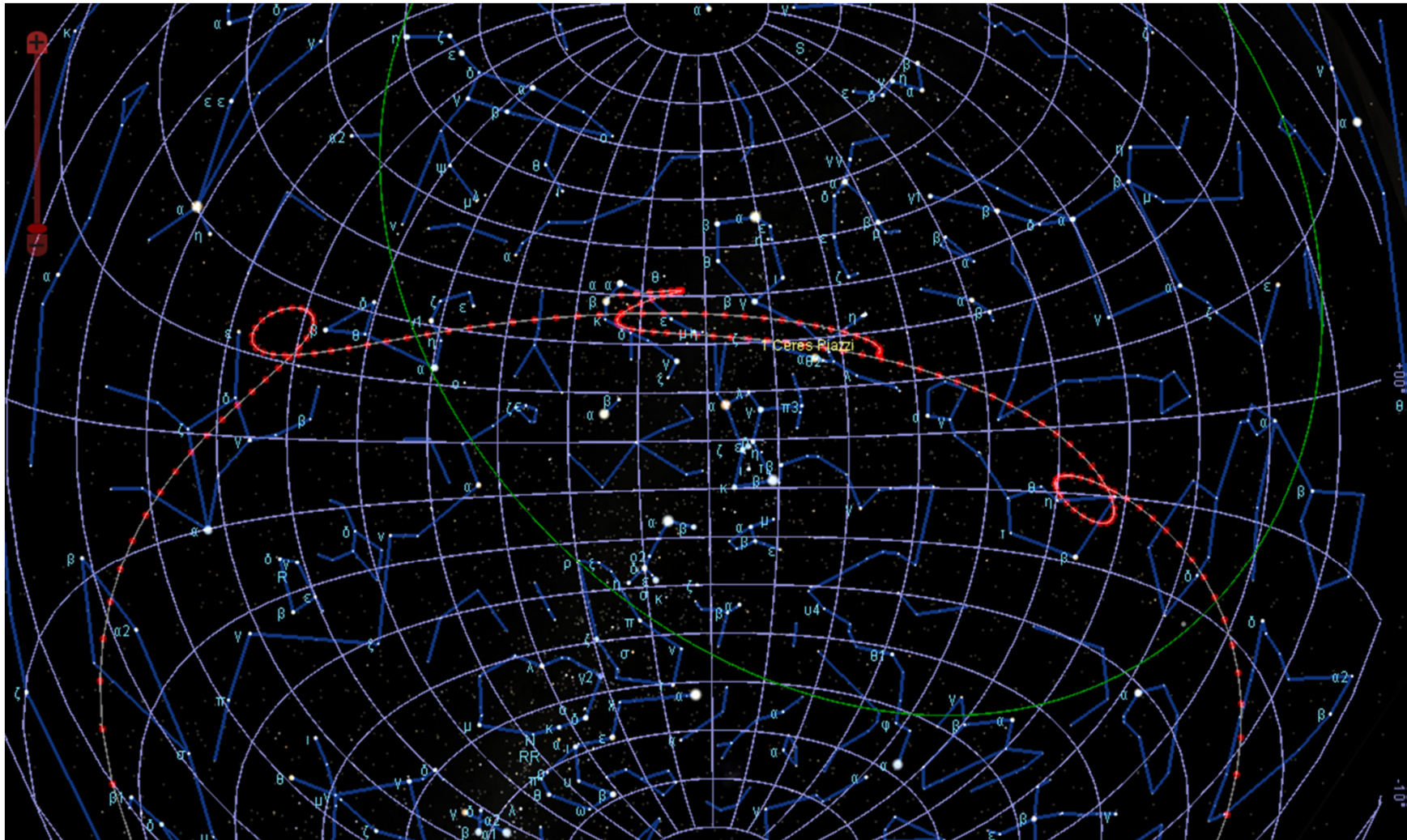
- Given n ≥ 3 observations, differentially corrects Der IOD or HGM initial state estimate
- Uses UPM (Uniform Path Mechanics) to propagate orbit of arbitrary eccentricity (i.e., UPM is a "universal variables" method)

Prepared for the AMOS Conference held September 20-23, 2016 in Maui, Hawaii USA. For further information, visit <https://amos.tech.com/amos-technical-papers/proceedings/>

Summary of Findings

1. Out of 19 complete Piazzi observations, only two were assessed as "bad" (1801 Jan 3 and Jan 11).
2. Ephemeris propagated from contemporary Batch UPM DC solution with 17 Best Piazzi observations was slightly worse than Gauss's 1801 December search ephemeris.
3. Ephemeris propagated from contemporary Batch UPM DC solution with 3 Piazzi observations* was slightly better than Gauss's 1801 December search ephemeris. (*Used exact same 3 observations that Gauss himself used: 1801 Jan 1, Jan 21, and Feb 11.)

Path of Ceres during 1801-1802



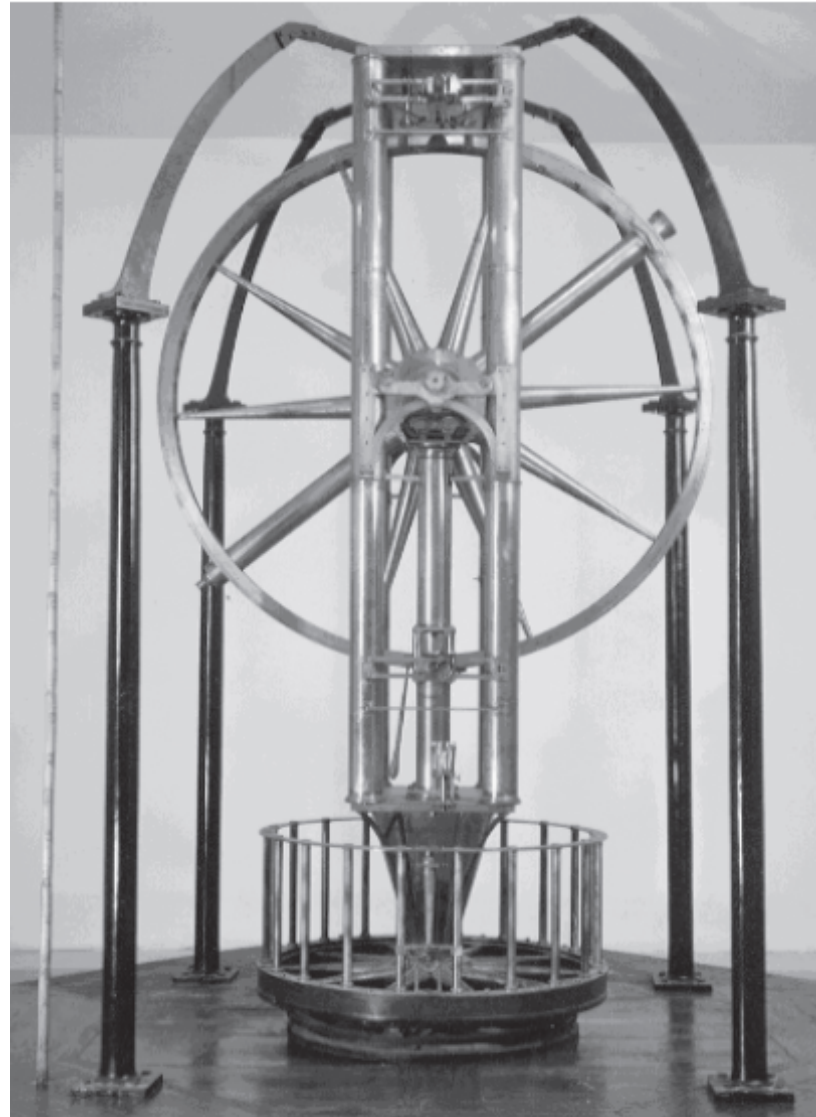
Ceres was in the “shoulder” of Taurus when Piazzi discovered it the night of 1801 January 1.
As the year progressed, Ceres moved through Gemini and Cancer and into Leo.
In early 1802 Ceres looped while heading into Virgo.
(Graphic produced via Software Bisque’s TheSkyX)



Timeline 1801-1802

- **Jan 1 - Feb 11 1801**
 - Piazzi discovers Ceres at Palermo Observatory

The Palermo Circle (at right). This meridian circle was constructed by Jesse Ramsden (1730–1800), the greatest of the eighteenth-century instrument makers. It was completed in 1789 after almost two years of intense work. The telescope has a 7.5-cm objective lens; the altitude scale (5 feet in diameter) was read with the aid of two diametrically-opposed micrometer microscopes while the azimuth scale (3 feet in diameter) was read by means of a micrometer microscope.



Timeline 1801-1802



- September 1801

- Von Zach publishes Piazzì's observations in *Monatliche Correspondenz*

Beobachtungen des zu Palermo d. 1. Jan. 1801 von Prof. Piazzì neu entdeckten Gestirns.

1801	Mittlere Sonnen- Zeit	Gerade Aufstieg in Zeit	Gerade Auf- steigung in Grad.	Nördl. Abweich.	Geocentri- sche Länge	Geocentr. Breite	Ost der Sonne + 20" Aberration	Logar. d. Distanz ☉ ♂
	St. "	St. "	" "	" "	Z "	" "	Z "	
Jan.	1 8 43 17,8	3 27 11,25	51 47 48,8	15 37 43,5	1 23 22 58,3	3 6 42,1	9 11 1 30,9	9,9926156
	2 8 39 14,6	3 26 53,85	51 43 27,8	15 41 5,5	1 23 19 44,3	3 2 24,9	9 12 2 18,6	9,9926317
	3 8 34 53,3	3 26 38,4	51 39 36,0	15 44 31,6	1 23 16 58,6	2 58 9,9	9 13 3 16,6	9,9926324
	4 8 30 42,1	3 26 23,15	51 35 47,3	15 47 57,6	1 23 14 15,5	2 53 55,6	9 14 4 14,9	9,9926418
	10 8 6 15,8	3 25 32,1	51 28 1,5	16 10 32,0	1 23 7 59,1	2 29 0,6	9 20 10 17,5	9,9927641
	11 8 2 17,5	3 25 29,73	51 22 26,0
	13 7 54 26,2	3 25 30,30	51 22 34,5	16 22 49,5	1 23 10 27,6	2 16 59,7	9 23 12 13,8	9,9928490
	14 7 50 31,7	3 25 31,72	51 22 55,8	16 27 5,7	1 23 12 1,2	2 12 56,7	9 24 14 13,5	9,9928809
	17	16 40 13,0
	18 7 35 11,3	3 25 55, ..	51 28 45,0
	19 7 31 28,5	3 26 8,15	51 32 2,3	16 49 16,1	1 23 25 59,2	1 53 38,2	9 29 19 53,8	9,9930607
	21 7 24 2,7	3 26 34,27	51 38 34,1	16 58 35,9	1 23 34 21,3	1 46 6,0	10 1 20 40,3	9,9931434
	22 7 20 21,7	3 26 49,42	51 42 21,3	17 3 18,5	1 23 39 1,8	1 42 28,1	10 2 21 32,0	9,9931886
	23 7 16 43,5	3 27 6,90	51 46 43,5	17 8 5,5	1 23 44 15,7	1 38 52,1	10 3 22 22,7	9,9932348
28 6 58 51,3	3 28 54,53	52 13 38,3	17 32 54,1	1 24 15 15,7	1 21 6,9	10 8 26 20,1	9,9935062	
30 6 51 52,9	3 29 48,14	52 27 2,1	17 43 11,0	1 24 30 9,0	1 14 16,0	10 10 27 46,2	9,9936332	
31 6 48 26,4	3 30 17,25	52 34 18,8	17 48 21,5	1 24 38 7,3	1 10 54,6	10 11 28 28,5	9,9937007	
Febr.	1 6 44 59,9	3 30 47,2	52 41 48,0	17 53 36,3	1 24 46 19,3	1 7 30,9	10 12 29 9,6	9,9937703
	2 6 41 35,8	3 31 19,06	52 49 45,9	17 58 57,5	1 24 54 57,9	1 4 1,5	10 13 29 49,9	9,9938423
	5 6 31 31,5	3 33 2,70	53 15 40,5	18 15 1,0	1 25 22 43,4	0 54 23,9	10 16 31 45,5	9,9940751
	8 6 21 39,2	3 34 58,50	53 44 37,5	18 31 23,2	1 25 53 29,5	0 45 5,0	10 19 33 33,3	9,9943276
	11 6 11 58,2	3 37 6,54	54 16 38,1	18 47 58,8	1 26 26 40,0	0 36 2,9	10 22 35 11,4	9,9945823



Timeline 1801-1802

- **October - November 1801:**

- von Zach publishes orbital elements and search ephemerides
- Johann Burckhardt, Wilhelm Olbers, and Giuseppe Piazzi all contribute analyses
- But it is the young (age 24) contributor, Dr. Carl Friedrich Gauss of Braunschweig (Brunswick), whose analyses prove most promising

- **Essential reading:**

- Von Zach, Franz Xaver, *Monatliche Correspondenz zur Befoerderung der Erd- und Himmelskunde, Vol. 4*, Nabu Public Domain Reprint (in German)
 - All articles cited on this timeline for 1801-1802 can be found there
- “Giuseppe Piazzi and the Discovery of Ceres,” lead article by G. Fodera Serio, A. Manara, and P. Sicoli in *Asteroids III*, edited by W.F. Bottke, Jr., A. Cellino, P. Paolicchi, and R.P. Binzel, as published by University of Arizona Press (Tucson, 2002)
 - This particular article in the 785-page book *Asteroids III* can be found online
- *Discovery of the First Asteroid, Ceres: Historical Studies in Asteroid Research*, by Clifford Cunningham (Springer, 2016)
 - Good source for further historical information about Piazzi, von Zach, Burckhardt, Olbers, Gauss, and others (e.g., Piazzi’s assistant, Niccolo Cacciatore)



Timeline 1801-1802

- **December 1801:**

- von Zach publishes Ceres orbital elements and search ephemeris of Gauss
- Von Zach recovers Ceres on night of 1801 Dec 31 - 1802 Jan 1 using search ephemeris of Gauss

Search Ephemeris of Gauss
from *Monatliche Correspondenz*,
Vol. 4. p. 647:

**Aus diesen Elementen hat Dr. Gauss folgende
Orter der Ceres Ferdinandea im voraus berechnet.
Die Zeit ist mittlere für Mitternacht in Palermo.**

1801	Geocentrische Länge	Geocentrische Breite nördl.	Logarith. des Abstandes von der ☉	Logarith. des Abstandes von der ☾	Verhältniß der gefundenen Helligk.
	Z				
Nov. 25	5 20 16	9 25	0,42181	0,40468	0,6102
Dec. 1	5 22 15	9 48	0,40940	0,40472	0,6459
	7 5 24 7	10 12	0,39643	0,40479	0,6835
	13 5 25 51	10 37	0,38296	0,40488	0,7290
	19 5 27 27	11 4	0,36902	0,40499	0,7770
	25 5 28 53	11 32	0,35468	0,40512	0,8295
	31 6 0 10	12 1	0,34000	0,40528	0,8869

Z column contains "Zodiac Number" 0 through 11, to be multiplied by 30 degrees and added to degrees column

Table 4, Page 11 of my AMOS 2016 paper converts Gauss's **geocentric ecliptic longitudes and latitudes to right ascensions and declinations***

Gregorian Date	Ecliptic Longitude*	Ecliptic Latitude	Right Ascension	Declination
year mo da	deg mn	deg mn	hours	degrees
1801 11 25	170 16	09 25	11.6558	12.5032
1801 12 01	172 15	09 48	11.7885	12.0665
1801 12 07	174 07	10 12	11.9141	11.6897
1801 12 13	175 51	10 37	12.0316	11.3805
1801 12 19	177 27	11 04	12.1417	11.1550
1801 12 25	178 53	11 32	12.2418	11.0116
1801 12 31	180 10	12 01	12.3331	10.9438

*Using formulas in Chapter IV of William Marshall Smart's, *Text-Book on Spherical Astronomy*, 5th edition (Cambridge University Press, 1965), p. 40.



Timeline 1802 - Present

- **Piazzi's goal as a mathematician and astronomer was to compile a star catalog**
 - for the purpose of being able to tell when something new appeared in the night sky, e.g., a new comet or a new planet
 - Piazzi's efforts were toward achieving what we now call "space situational awareness"
- **So I thought it important to allude to progress with star catalogs since 1801, in that:**
 - Star catalogs are very important today for precise deep-space object position determination (astrometry)
 - They are used to "register" the streaks or points of light against the RA-DEC coordinate grid
- **Sub-arcsecond astrometric RA-DEC position determination becomes possible with accurate star catalogs**
 - As good as the mechanical pointing of the Software Bisque Paramounts is (± 30 arcsec), it is not sufficient for high-accuracy, angles-only (RA-DEC) observation collection
 - That is, one needs a high-precision star catalog and star registration software to achieve sub-arcsecond astrometry



- **Anecdote**

- One of my tasks as electro-optical systems engineer on GEODSS was to develop an algorithm for star selection with the new (in 2012) SSTRC1* star catalog of the U.S. Naval Observatory (**Space Surveillance Telescope Reference Catalog 1*)
- This SSTRC1 catalog has 378 million stars, but we only needed about 1.2 million stars for the GEODSS registration star catalog, wherein we wanted approximately 100 stars per field of view (FOV = 1.6 degrees horizontal by 1.2 degrees vertical)
- To address this task, I came up with the concept of "spherical rectangles"
 - Showed that the entire celestial sphere is a spherical rectangle
 - In so doing, was able to come up with a mathematical algorithm for generating subcatalogs of any specified size, subcatalogs that provide "uniform areal density"
 - I called this algorithm the TIER method (Tesselate Into Equal-area subRectangles).
- I told Steve Bisque about my TIER method at AMOS 2016
 - Steve told me that he has his own algorithms for generating subcatalogs of uniform areal density, algorithms that he developed years ago
 - Of course!
 - Another reason that *TheSky* is to me such an amazing body of software



The Contemporary Algorithms

- **Der IOD (Initial Orbit Determination)**
 - $n = 3$ angles-only observations (topocentric RA, DEC)
 - Long arcs admissible, up to one orbital revolution
 - Solves for range via 8th degree polynomial
 - Epoch of elements is at middle observation time
- **HGM IOD – Heliocentric motion**
 - HGM is acronym for “Herget/UPM” (UPM = Uniform Path Mechanics)
 - n angles-only observations (topocentric RA, DEC)
 - $n \geq 3$ but subject to short-arc limitation
 - Iterates on range guesses for first and last observations
 - Epoch of elements is at first observation time
- **ORBIT2 Numerical Integration**
 - Numerically integrates equations of motion of major Solar System planets and “Spare”
 - “Spare” can be space probe, comet, asteroid, or dwarf planet
 - Starts with DE405 initial state vectors of planets and J2000 initial state vector for “Spare”
- **Batch UPM DC – Heliocentric motion**
 - Given $n \geq 3$ observations, differentially corrects Der IOD or HGM initial state estimate
 - Uses UPM, a “universal variables” method, to propagate orbit of arbitrary eccentricity e .
 - The UPM procedure does not need to test for and branch on $e < 1$ (elliptical), $e = 1$ (parabolic), or $e > 1$ (hyperbolic) orbits/paths/trajectories



Batch UPM DC (excerpted from the paper)

Batch UPM DC (the heliocentric version is referred to as "HDC") is documented for artificial Earth satellites in [5], [8], [12], and [13]. Given n observations consisting of two measurements each (topocentric right ascension and declination), the basic vector-matrix equation of batch least squares estimation is

$$\mathbf{X}_0' = \mathbf{X}_0 + (\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{W} [\mathbf{Y} - \mathbf{F}(\mathbf{X}_0)]$$

where \mathbf{X}_0' is the updated 6-by-1 state estimate (position and velocity at epoch), \mathbf{X}_0 is the previous or initial state estimate to be iterated, and \mathbf{Y} is the measurements vector with $N = 2n$ components consisting of the topocentric right ascension and declination measurement quantities

$$\cos(\delta) \Delta\alpha \quad \text{and} \quad \Delta\delta .$$

\mathbf{A} is the N -by-6 matrix of partials of the components of $\mathbf{F}(\mathbf{X})$ with respect to the components of \mathbf{X}_0 and \mathbf{W} is the N -by- N diagonal weight matrix consisting of the reciprocals of the variances in the measurements. (For the purposes of this application, \mathbf{W} is the identity matrix.) It is seen then that $[\mathbf{Y} - \mathbf{F}(\mathbf{X}_0)]$ is the residuals vector for the current iteration.



More about Herget/UPM

- **HGM employs n observations ($n \geq 3$)**
 - “Classical” initial orbit determination (IOD) methods use exactly three
 - If $n = 3$, HGM provides an exact, two-body fit
- **HGM is modular with respect to the Lambert solution**
 - HGM is “short-arc,” but HGM could incorporate any/all of the Lambert solutions of Gauss, Battin, Gooding, and Der
- **HGM is a two-parameter fit of the first and last topocentric distances to the intermediate observations**
 - HGM is a standalone IOD method (does not need a “seed” estimate of state to start)
- **HGM solution can serve as a “seed” to a batch differential correction (DC) that performs a six-parameter (or more) fit**
 - HGM provides a two-parameter fit, then
 - Batch DC, e.g., Batch UPM DC, provides a six-parameter fit of position and velocity at epoch to all of the available observations. Therefore:
 - HGM and Batch UPM DC constitute a two-application sequence that provides the best possible fit, in the least-squares sense, of position and velocity at epoch to all of the available observations



Notes on Fundamental Algorithms

- **The Contemporary Algorithms are representative of a suite of fundamental algorithms in dynamical astronomy and astrodynamics**
 - These fundamental algorithms apply whether the secondary object in motion about its primary is an Earth satellite, a missile, an interplanetary space probe, a planet, or an asteroid
 - The suite is: Orbit Propagation, Preliminary Orbit Determination, and Differential Correction
- **Orbit Propagation is also known as the “Kepler Problem”**
 - Given secondary’s position and velocity at some time t_0 , determine its positions and velocities (in a reference frame defined by the primary) at some earlier or later times t_1, t_2, \dots, t_n
 - ORBIT2 solves the Kepler problem simultaneously for the Sun, eight major planets, and the body called “Spare”. That is,
 - ORBIT2 integrates twenty-seven second-order, ordinary differential equations simultaneously
- **Preliminary Orbit Determination is also called Initial Orbit Determination (IOD)**
 - Assumes two-body mechanics (secondary and primary, e.g., asteroid and Sun)
 - **Lambert IOD Problem:** Given two position vectors of the secondary and the time of flight from the first to the second, determine the velocity needed at the first position for the secondary to fly to the second position in the specified amount of time (used in processing radar obs)
 - **Angles-Only IOD Problem:** Given three angles-only observations of the secondary, e.g., three of (time, right ascension, declination), but no corresponding range (distance) measurements, determine the velocity required at the (typically) first or middle observation time, so as to be able to calculate the exact ranges of the secondary at each of the three observation times (used in processing electro-optical [telescopic] observations)



Notes on Fundamental Algorithms, II

- In solving the Angles-Only IOD problem for Ceres using three appropriately-spaced Piazzi observations, Gauss also solved the Lambert problem
- Gauss derived the process that we now call Differential Correction (see Gauss's *Theoria Motus*, published in 1809) and employed it, at least in principle, in his calculations for Ceres
- **Differential Correction (DC) is used to improve the Preliminary Orbit**
 - A simple example of *linear* least squares parameter estimation is the procedure by which, given more than two (x,y) points on a straight line, we find the slope and y -intercept of the line that minimizes the y -distances from the three or more (x,y) points to the corresponding (x,y_{line}) points on the line (the distances $y - y_{line}$ are called the residuals)
 - Differential correction, as a nonlinear least squares process, generalizes the linear case
 - An essential feature of *nonlinear* least squares is that the solution equations must be iterated until convergence is obtained, wherein the square root of sum of squares of residuals decreases to its lowest possible value from iteration to iteration
 - The iterative process must be initiated via an initial estimate of position and velocity obtained from a preliminary orbit determination method such as Der IOD or HGM
 - But differential correction is much more powerful than IOD, in that, in, say, the heliocentric case, the DC typically models gravitational perturbations of the secondary's orbit by bodies other than the Sun
- **Statistical Orbit Determination**
 - Differential Correction is also called "Nonlinear Least-Squares Orbit Determination" and can be generalized even further to "Statistical Orbit Determination"
 - This leads to the concept of filters e.g., extended Kalman filters and "unscented" Kalman filters, which are very important in aerospace and mechanical engineering



Flow of Solutions and Comparisons

- **Ran HGM and Der IOD**
 - HGM with all 19 Piazzi observations
 - Der IOD with same 3 observations that Gauss used
 - Both preliminary solutions agreed, are valid, and worked to seed (initiate) Batch UPM DC
- **Ran Batch UPM DC with all 19 Piazzi observations**
 - Fig. 6 shows that obs 3 and 6 spoil the fit. Fig. 7 shows results with obs 3 and 6 rejected.

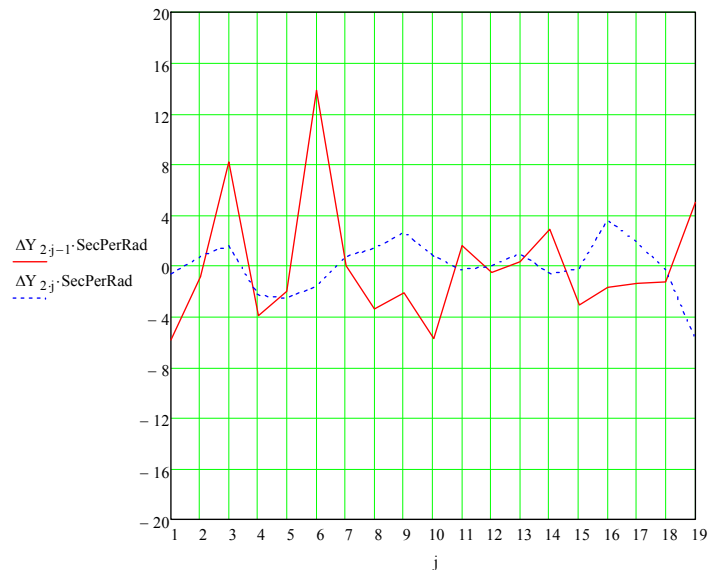


Fig. 6. Plot of HDC RA and DEC Residuals for n=19 Actual Piazzi Observations. Note that observations 3 and 6 spoil the fit. (Red colors right ascension residuals plot and blue colors declination residuals plot. Vertical axis units are arc-seconds.)

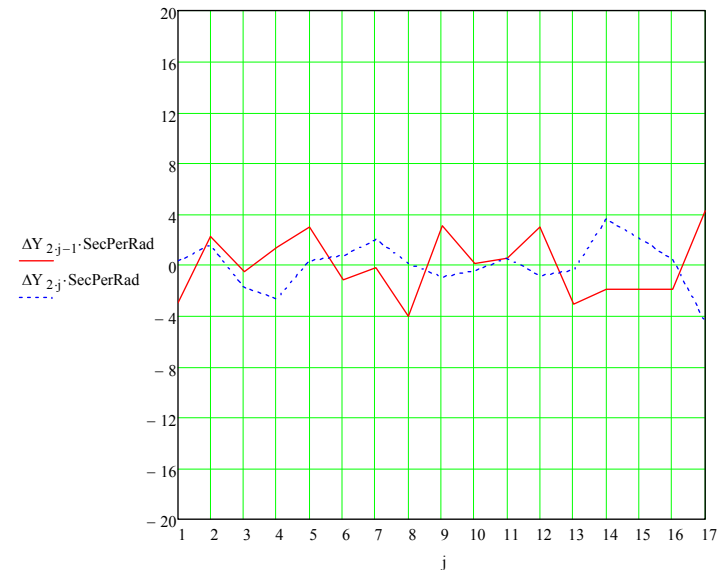
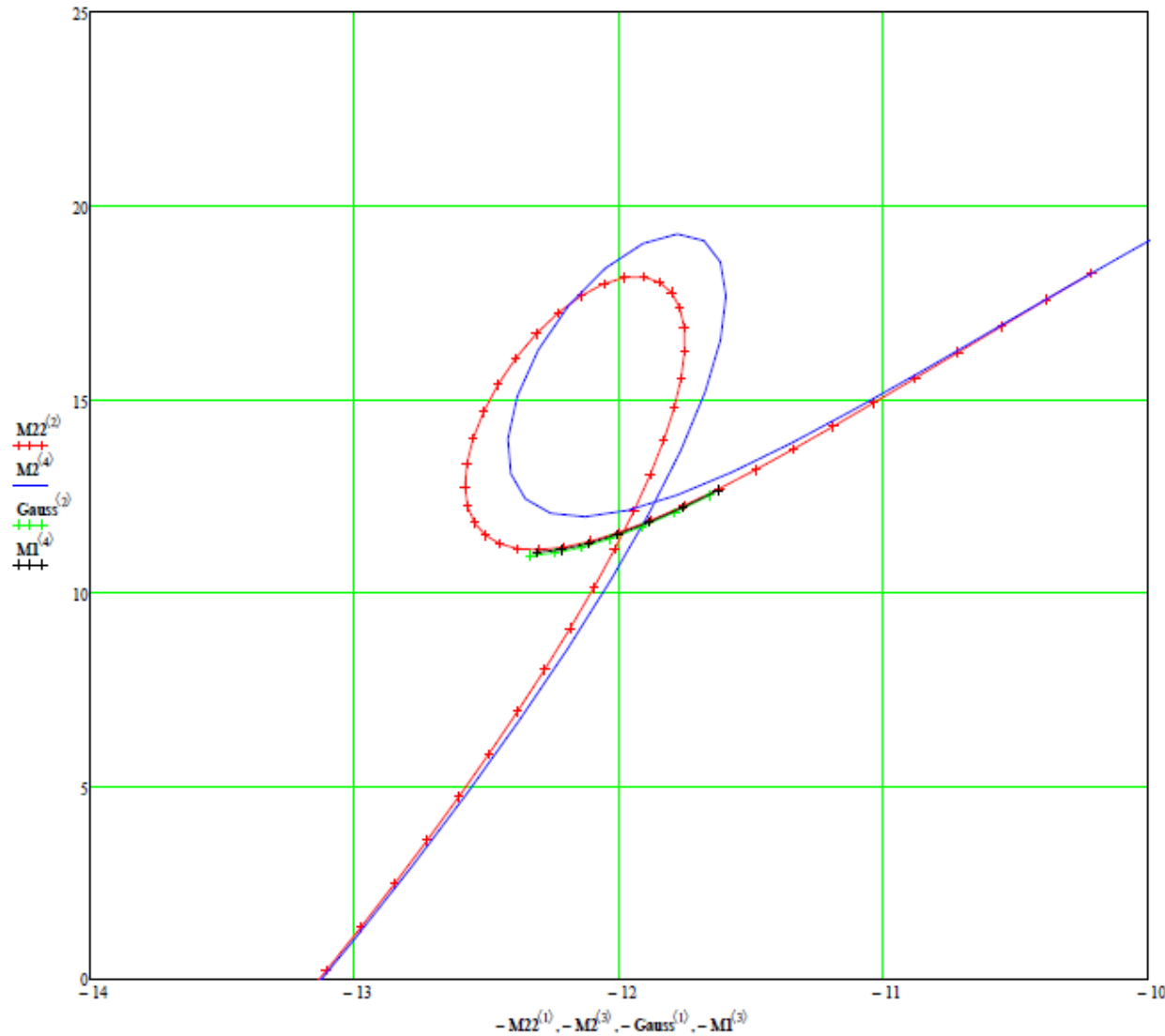


Fig. 7. Plot of HDC RA and DEC residuals for n=17 Best Actual Piazzi Observations. The best 17 actual Piazzi observations are the 17 observations remaining from the original set of 19 observations after obs 3 and 6 were deleted.



- **Ran Batch UPM DC with 17 Best Piazzi observations**
 - Fig. 7, previous slide, showed DC's final-pass residuals with obs 3 and 6 rejected
 - All residuals now within +/- about 4 arcseconds for converged iteration (final pass)
 - Used resulting solution to plot RA-DEC points at the same time points as in Gauss's search ephemeris from December 1801
 - Also plotted the points from Gauss's search ephemeris and from the ORBIT2 predictions
 - Gauss's results agreed with ORBIT2 better than contemporary solution with 17 best Piazzi observations
 - That is, Batch UPM DC solution with 17 best Piazzi obs was *slightly worse than* Gauss's!
 - How could this be? To investigate further, did the following:
- **Ran Batch UPM DC with exact same 3 obs that Gauss used**
 - Now contemporary Batch UPM DC with just these three Piazzi obs was *slightly better than* Gauss's solution from 1801
 - Meantime, was able to find JPL Horizons website, use with Ceres, and see that the JPL Horizons calculations for Ceres are better than those that I was able to obtain with ORBIT2
 - Why better?
 - ORBIT2 numerically integrated all major planets and Spare (Ceres) from 1997 back to 1801. It was never intended to go that far forward or backward from J2000 without differential correction, whereas
 - JPL Horizon ephemeris predictions are based upon a differential correction using all available historical observations
- **End result of all this analysis was the plots on the next slide**

Flow of Solutions and Comparisons, III



Plot of JPLHorizons,
ORBIT2, Gauss Search Ephemeris,
and "Batch UPM DC Solution with 3
Observations" Sky Traces for Ceres
1801-1802

Legend

M2⁽²⁾ is JPL Horizons ephemeris (red).

M2⁽⁴⁾ is ORBIT2 ephemeris (blue).

Gauss is Gauss search ephemeris
(green).

M1 is Batch UPM DC ephemeris based
upon 3-observation DC solution (black).

This plot shows that the ORBIT2 sky
trace (blue) closely follows JPL Horizons
sky trace (red) except where the traces
loop, starting at end of 1801 and on into
1802.

It also shows that the **M1** curve points
are closer to the corresponding **M2⁽²⁾** and
M2⁽⁴⁾ curve points than are the
corresponding **Gauss** curve points. In
other words, the contemporary Batch
UPM DC-based solution is slightly better
than the 1801 Gauss search ephemeris
solution. This improvement is believed to
be due to our contemporary solar
ephemeris model in 2016 being better
than the solar ephemeris model that was
available to Gauss in 1801.



Summary of Findings

- **Of Piazzi's 19 complete observations from 1801, all were quite good**
 - But obs 3 and 6 were not quite as good as the rest, so they were rejected
- **The DC with the 17 best Piazzi observations was very good**
 - But not quite as good as Gauss's solution with 3 hand-selected observations
- **Hand-selecting the three best observations yielded the best "DC"**
 - Note that "least squares" statistics did not come into play here. That is:
 - Six independent measurements, from three hand-selected observations, were used to determine six independent quantities, the state vector of Ceres (position and velocity) on 1801 January 1
- **Gauss was fortunate that Piazzi's observations were so good**
 - But Gauss not only developed an IOD procedure that we still use today,
 - Gauss also demonstrated, by the example of Piazzi's observations, that there are situations where a judicious choice of just three observations (the minimum number required) can yield a better solution than a statistically-valid solution based upon the entire set of available observations
 - We are left to marvel again, not only at Gauss's role in the discovery and recovery of the first of a new class of solar system bodies (the asteroids),
 - but also at the calculations that Gauss was able to accomplish 217 years ago with just quill pen, ink, paper, and logarithm tables



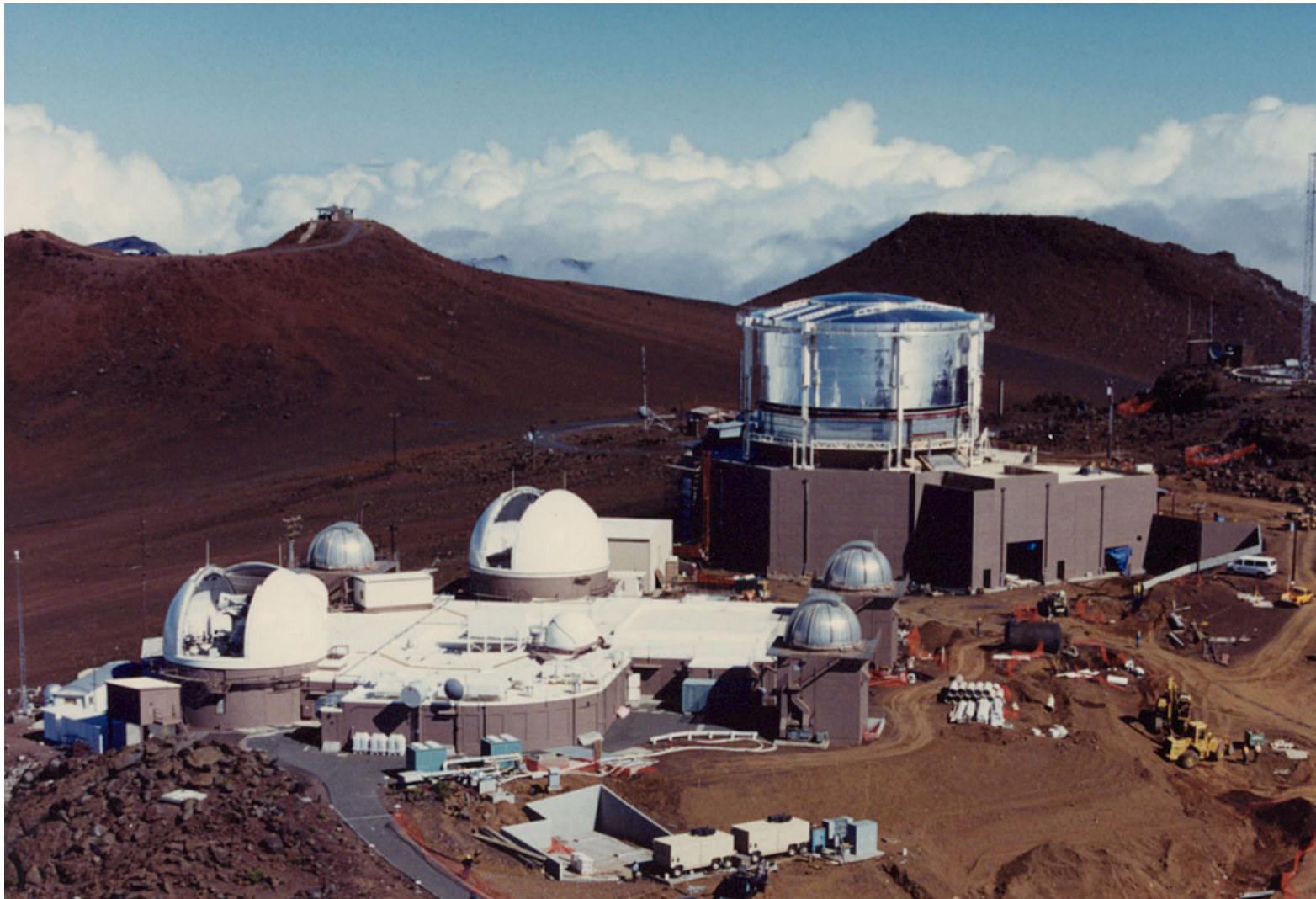
- **Batch UPM DC algorithm and numerical calculations are available at astroger.com webpage**
 - Implemented in Mathcad and provided as downloadable .pdf files
 - There are two worksheets, the “initiator” and the “iterator”
 - The 17 best Piazzi observations are used in the DC
 - Every equation is there, every sine, cosine, and arctangent, along with explanations and numerical results
 - If you have Mathcad, you can email me to request the Mathcad input and .xmcd files
- **von Zach's December 1801 article in *Monatliche Correspondenz* is also at the astroger.com webpage**
 - Pages 638-649, including Gauss's search ephemeris on p. 647
 - The text is in German
 - May take awhile to download if you have a slow internet connection
- **And finally, also at astroger.com**
 - A summary translation of von Zach's December 1801 article in *Monatliche Correspondenz*
 - And a very brief history of the discovery and recovery of Ceres



The AMOS Conference in Maui

- **Held annually in September**
 - Currently held at Marriott Wailea Beach Resort & Spa (a great venue)
 - All technical papers are individually available online as downloadable PDF files
 - See <https://amostech.com/> for 2017 conference plans. This link includes highlights of the 2016 conference
- **The acronym “AMOS” has had several definitions over the years**
 - AMOS originally stood for “ARPA Midcourse Observation Station”, where ARPA = Advanced Research Projects Agency (now DARPA , i.e., DARPA = Defense ARPA)
 - In 1977 Strategic Air Command assumed control of the AMOS site and renamed it to Air Force Maui Optical Station (AMOS)
 - Latest name is “Air Force Maui Optical and Supercomputing Site”
 - But the AMOS conference itself is called the “Advanced Maui Optical and Surveillance Technologies” conference
- **The AMOS conference originated due to activities supporting the ongoing Air Force electro-optical space mission on Mt. Haleakala**
 - 2016 was the 50th year of the Air Force space mission on Maui
 - The AMOS 2016 conference had the largest attendance ever – 686 participants
 - Because of the number of telescopes and facilities on Mt. Haleakala, it makes sense to hold the conference in Maui

USAF Facilities on Mt. Haleakala



The Air Force Maui Optical and Supercomputing site at [Haleakala Observatory](#) in Hawaii. Facilities shown include the Advanced Electro-Optical Telescope, the Maui Space Surveillance System, and one of three Ground-based Electro-Optical Deep Space Surveillance sites.



- **This presentation about**
 - Ceres (an asteroid)
 - Piazzi (a skilled but lucky observer)
 - Von Zach (a pioneering astronomical journal publisher)
 - Gauss (mathematical prodigy and genius)
 - Orbit Propagation (ways to generate ephemerides)
 - Preliminary Orbit Determination (ways to characterize orbits and initiate DCs)
 - Differential Correction (ways to improve/update orbits – the precursor to modern Statistical Orbit Determination)
- **will conclude with**
 - a brief slide show, followed by
 - Questions and answers



END NOTES



About my co-author, Dr. Gim. J. Der

Dr. Gim J. Der did his undergraduate work at the University of London and began his doctoral studies under Ted Edelbaum at MIT in Cambridge, Massachusetts. When Dr. Edelbaum died unexpectedly, Dr. Richard H. Battin took over as Gim's thesis advisor. In my own estimation, Gim's multi-rev, perturbed Lambert algorithm and his angles-only IOD algorithm are "world-class."

Richard Battin recently died, in 2014. He was author of two books, *Astronautical Guidance* and *An Introduction to the Mathematics and Methods of Astrodynamics*. Battin was the principal architect of the Apollo guidance, navigation, and control algorithms and software.

Although there is no official ranking that I know of, I believe that Dr. Battin was considered by the guidance and control community to be the top astrodynamist in the world after the highly successful Apollo missions to the Moon.

Now in addition to his being mentored by Dr. Battin, Gim was also a protege of the eminent dynamical astronomer and astrodynamist John P. Vinti, also of MIT. Before Dr. Vinti died, he gave Gim the unfinished manuscript of his book, *Orbital and Celestial Mechanics*. Gim Der and Nino Bonavito completed Vinti's book and it was published by the AIAA in 1998 (show the book during live presentation).

Gim Der and I have collaborated off and on now since fall 2014. Our collaboration so far has resulted in two papers by Dr. Der at the AAS/AIAA Astrodynamics Specialist Conference in Vail during August 2015, and this paper by me.

It was Gim's idea for me to do my paper for AMOS 2016. The AMOS 2016 paper came about just before Christmas 2015 when Gim said to me, "Roger, since we have been working together for over a year now, I have written two papers, and it seems to me that you ought to be able to do a paper on what you have done as part of our collaboration. So you write a paper now. Put me on as co-author. I will review and edit it."

My initial idea was to submit an abstract for the AIAA conference in Long Beach, California, held in August 2016, since Gim lives in the Los Angeles area. But Gim said, "No, let's go to Hawaii for this. Submit an abstract for AMOS 2016. I have presented there twice now, in 2011 and 2012. AMOS is a more relaxed and less formal conference."



About my Denver Friend and Colleague, Dr. Robert E. Stencel

I first met "Dr. Bob," as we his Denver area followers affectionately call him, at the American Astronomical Society meeting in Denver in 2004. At Dr. Bob's invitation, I have visited the Meyer-Womble Observatory on Mt. Evans, and have overnighted at the Echo Lake sleeping facility close by more than one once (if I remember correctly, one time I brought up Cincinnati chili that I had cooked for dinner). Dr. Bob even let me lecture on orbital mechanics in one of his University of Denver courses.

Dr. Bob introduced me in 2005 to Dr. John Mather. Dr. Mather was a principal on NASA's highly successful Cosmic Background Explorer (COBE) mission, and author of *The Very First Light*. One year later, in 2006, Dr. Mather was awarded the Nobel Prize in Physics for showing, via COBE, that the universe behaves today like a perfect blackbody radiating at a temperature of 2.725 Kelvins. This, it has been shown, supports the theory that the universe has aged 13.7 billion years since the Big Bang.

Dr. Bob invited me to activities relating to his research on the variable star, epsilon Aurigae, during its minimum of 2009-2011. I translated from German to English for him Hans Ludendorff's 1904 and 1912 research papers on this previously enigmatic star.

Dr. Bob introduced me to the Bisque brothers (Steve, Tom, Daniel, and Matthew) in early 2006. During that year, I developed for Software Bisque a C++ program to predict the visibility of Iridium flares. Steve subsequently incorporated the code into *TheSkyX*.



About the Bisque Brothers and Software Bisque

Ever since spring 2006, spent in close collaboration with Tom Bisque while I was writing the Iridium flare prediction software, I have kept in touch.

It is a funny story that when, also in 2006, I brought Dr. TS Kelso to Golden to meet the Bisque brothers, they were in awe of him. We all had lunch at the Table Mesa Restaurant in Golden. They treated TS like a hero, like an oracle. Why? Because TS *is* an oracle! From military space to commercial space, almost everyone seems to know, or know of TS, his CelesTrak website, his AGI affiliation, and his vast knowledge of and connections in the space world.

So I am proud to say that TS was a student in my vector calculus classes that I taught at the U.S. Air Force Academy when TS was a second-year cadet.

Now I have to confess ... I myself am in awe of the Bisque brothers! Why? Because they have done so much to further the science of astronomy, as done with robotic telescopes and electro-optical imaging, via *TheSky* and their Paramount mounts.

And because I recently I watched Daniel Bisque's presentation to the Denver Astronomical Society about the history of Software Bisque. Therein I learned so much more about Software Bisque and their products than I never knew before.